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The MSW solution to the solar neutrino problem in the presence of random solar matter density perturbations

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We present the evolution equation describing MSW conversion, derived in the framework of the Schrödinger approach, in the presence of matter density fluctuations. Then we analyse the effect of such fluctuations in the MSW scenario as a solution to the solar neutrino problem. It is shown that the non-adiabatic MSW parameter region is rather stable (especially in δm^2) for matter density noise at the few percent level. We also discuss the possibility to probe solar matter density fluctuations at the future Borexino experiment.

1. The present deficit of solar neutrinos seems to disfavour any "astrophysical solutions" [2] whereas it points to neutrino oscillations. In particular the resonant conversion due to neutrino interactions with constituents of the solar material (the Mikheyev-Smirnov-Wolfenstein (MSW) effect) [3] is the most elegant and viable explanation for the existing solar neutrino data. It provides an extremely good data fit in the small mixing region with $\delta m^2 \simeq 10^{-5}\mathrm{eV}^2$ and $\sin^2 2\theta \simeq 10^{-3} \div 10^{-2}$ [4,5,8].

Here we investigate the stability of the MSW solution with respect to the possible presence of random perturbations in the solar matter density.

The existence of matter density perturbations at the level of 1% or so cannot be excluded either by the Standard Solar Model (SSM), which is based on hydrostatic evolution equations, or by the present helioseismology observations [11].

Let us remind that in Ref.[7] the effect of periodic matter density perturbations added to an average density ρ_0, i.e. $\rho(r) = \rho_0[1+h\sin(\gamma r)]$ upon resonant neutrino conversion was investigated. In that case parametric resonance in the neutrino conversion can occur when the fixed frequency (γ) of the perturbation is close to the neutrino oscillation eigen-frequency and for rather large amplitude ($h \sim 0.1 - 0.2$). There are also a number of papers which address similar effects by different approaches [9,10].

In the present discussion we consider "white noise" matter dentity perturbations $\delta\rho$ (as in Ref. [10]). Namely we assume that the random field $\delta\rho$ has a δ-correlated Gaussian distribution:

$$\langle \delta\rho(r_1)\delta\rho(r_2)\rangle = 2\rho^2\langle\xi^2\rangle L_0\delta(r_1 - r_2) \tag{1}$$

where $\xi = \delta\rho/\rho$ and the correlation length L_0 obeys the following relation:

$$l_{free} \ll L_0 \ll \lambda_m \tag{2}$$

In (2) the lower bound is dictated by the hydrodynamical approximation used later on, $l_{\mathrm{free}} = (\sigma n)^{-1}$ being the mean free path of the particles in the solar matter [1]. On the other hand, the upper bound expresses the fact that the scale of fluctuations have to be much smaller than the characteristic ν matter oscillation length λ_m, as indeed the δ-correlated distribution in eq. (1) requires.

2. According to the standard Schrödinger equation approach, we derive now the most general neutrino evolution equation in random matter density. The evolution of a system of two

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[1]For Coulomb interactions, the cross-section σ is determined by the classical radius of electron $r_{0e} = e^2/m_e c^2 \sim 2\times 10^{-13}$cm, resulting in $l_{\mathrm{free}} \sim 10$cm for a solar mean density $n_0 \sim 10^{24}\mathrm{cm}^{-3}$.

neutrinos ν_e and ν_x ($x = \mu$ or τ)[2] in the solar matter is governed by

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} H_e & H_{ex} \\ H_{ex} & H_x \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}, \tag{3}$$

where the entries of the Hamiltonian matrix are

$$\begin{aligned} &H_e = 2[A_{ex}(t) + \tilde{A}_{ex}(t)], \quad H_x = 0, \\ &H_{ex} = \frac{\delta m^2}{4E}\sin 2\theta, \\ &A_{ex}(t) = \frac{1}{2}[V_{ex}(t) - \frac{\delta m^2}{2E}\cos 2\theta], \\ &\tilde{A}_{ex}(t) = \frac{1}{2}V_{ex}(t)\cdot\xi. \end{aligned} \tag{4}$$

Here θ is the neutrino mixing angle in vacuum, δm^2 the mass squared difference, E the neutrino energy and the matter potential for the $\nu_e \to \nu_x$ conversion reads

$$V_{ex}(t) = \frac{\sqrt{2}G_F}{m_p}\rho(t)(Y_e), \tag{5}$$

where m_p is the nucleon mass and Y_e is the electron number per nucleon.

The above system can be rewritten in terms of the following equations:

$$\begin{aligned} \dot{I}(t) &= H_e(t)R(t) - 2H_{ex}(t)(P(t) - 1/2) \\ \dot{R}(t) &= -H_e(t)I(t) \\ \dot{P}(t) &= 2H_{ex}(t)I(t), \end{aligned} \tag{6}$$

where $P = |\nu_e|^2$ is the ν_e survival probability, $R = \mathrm{Re}(\nu_x^*\nu_e)$ and $I = \mathrm{Im}(\nu_x^*\nu_e)$, with the corresponding initial conditions $P(t_0) = 1$, $I(t_0) = 0$, $R(t_0) = 0$.

The Eqs. (6) have to be averaged (see [1] for more details) over the random density distribution, taking into account that for the random component we have:

$$\langle \tilde{A}_{ex}^{2n+1} \rangle = 0, \quad \langle \tilde{A}_{ex}(t)\tilde{A}_{ex}(t_1) \rangle = 2\kappa\delta(t - t_1), \tag{7}$$

where the quantity κ is given by:

$$\kappa(t) = \langle \tilde{A}_{ex}^2(t) \rangle L_0 = \frac{1}{4}V_{ex}^2(t)\langle \xi^2 \rangle L_0. \tag{8}$$

[2]Here for simplicity we consider only the case of solar ν_e conversion into active state ν_μ or ν_τ; however the discussion can be extended also to the case of conversion into a sterile state [1].

In terms of the averaged quantities defined as $\langle P(t) \rangle = \mathcal{P}(\sqcup)$, $\langle R(t) \rangle = \mathcal{R}(\sqcup)$, $\langle I(t) \rangle = \mathcal{I}(\sqcup)$, we can write the variant of the set (6) as:

$$\begin{aligned} \dot{\mathcal{I}}(t) &= 2[A_{ex}(t)\mathcal{R}(\sqcup) - \kappa(\sqcup)\mathcal{I}(\sqcup) - \mathcal{H}_{\rceil\S}(\mathcal{P}(\sqcup) - \infty/\in)] \\ \dot{\mathcal{R}}(t) &= -2[A_{ex}(t)\mathcal{I}(\sqcup) + \kappa(\sqcup)\mathcal{R}(\sqcup)] \\ \dot{\mathcal{P}}(t) &= 2H_{ex}\mathcal{I}(\sqcup). \end{aligned} \tag{9}$$

This system of equations[3] explicitly exhibits the noise parameter κ. It is now possible to outline the main effects due to the presence of the random field $\delta\rho$ upon the resonant neutrino conversion. The MSW resonance condition remains unaltered, i.e. $A_{ex}(t) = V_{ex}(t) - \delta m^2 \cos 2\theta/2E = 0$, due to the random nature of the matter perturbations. Due to the condition $L_0 \ll \lambda_m$, the noise parameter κ (cfr. Eq.(8)) is always smaller than $A_{ex}(t)$ except at the resonance region. As a consequence, (see Eqs. (9)) the perturbation can show its maximal effect just at the resonance provided that the corresponding noise length $1/\kappa$ obeys the following *adiabaticity* condition at the resonance layer Δr

$$\tilde{\alpha}_r = \Delta r(\kappa)_{res} > 1. \tag{10}$$

This condition is analogous to the standard MSW *adiabaticity condition* at resonance $\alpha_r = \Delta r/(\lambda_m)_{res} > 1$ [3]. For definiteness, we have taken $L_0 = 0.1 \times \lambda_m$. The two adiabaticity parameters are related as

$$\tilde{\alpha}_r \approx \alpha_r \frac{\xi^2}{\tan^2 2\theta}, \qquad \alpha_r = \frac{\delta m^2 \sin^2 2\theta R_0}{4\pi E \cos 2\theta}. \tag{11}$$

Therefore due to the restriction (2) and for the range of parameters we are considering, $\xi \sim 10^{-2}$, $\tan^2 2\theta \geq 10^{-3} - 10^{-2}$, we have $\tilde{\alpha}_r \leq \alpha_r$.

As a result of (11), in the adiabatic regime $\alpha_r > 1$, the effect of the noise is enhanced to the extent that the mixing angle is small. Furthermore, the MSW non-adiabaticity $\alpha_r < 1$ always brings to $\tilde{\alpha}_r < 1$. As a result in our discussion the fluctuations are expected to be ineffective in the non-adiabatic MSW regime. Finally, it can be shown that the matter noise weakens

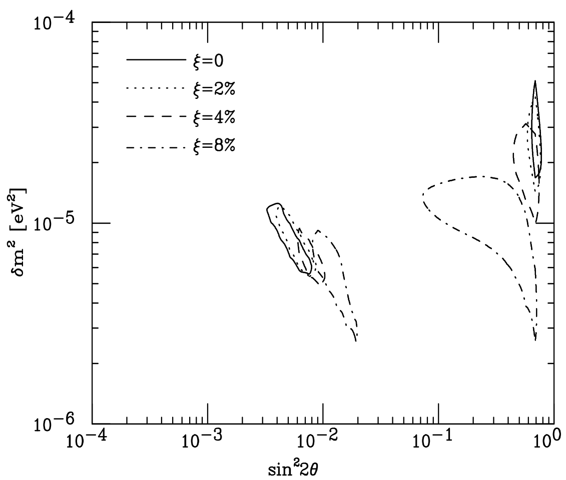
[3]These equations are equivalent to those obtained in Ref.[10] in terms of the variables $x = 2\mathcal{R}$, $y = -2\mathcal{I}$ and $r = 2\mathcal{P} - \infty$.

the MSW suppression in the resonance layer, exhibiting somehow the role of a friction.

3. In view of the qualitative features just outlined, we discuss the implications of noisy solar matter density in the MSW scenario for the solar neutrino problem. We have solved numerically the coupled differential Eqs. in (9) for the ν_e survival probability, using as reference SSM the most recent Bahcall-Pinsonneault model (BP) [6].

The χ^2 analysis has been performed taking the latest averaged experimental data of chlorine [12], gallium [13,14] and Kamiokande [15] experiments: $R_{Cl}^{exp} = (2.55 \pm 0.25)$SNU, $R_{Ga}^{exp} = (74 \pm 8)$SNU[4], $R_{Ka}^{exp} = (0.44 \pm 0.06) R_{Ka}^{BP}$.

Fig. 1



The results of the fitting in the $\delta m^2, \sin^2 2\theta$ parameter space, are shown in Fig. 1, where the 90% confidence level (C.L.) areas are drawn for different values of ξ. One can observe that the small-mixing region is almost stable, with a slight shift down of δm^2 values and a slight shift of $\sin^2 2\theta$ towards larger values.

The large mixing area is also pretty stable, exhibiting the tendency to shift to smaller δm^2 and $\sin^2 2\theta$. The smaller δm^2 values compensate for the weakening of the MSW suppression due to the presence of matter noise, so that a larger portion of the neutrino energy spectrum can be converted. The $\xi = 8\%$ case, considered for the sake of demonstration, clearly shows that the small mixing region is much more stable than the large mixing one even for such large value of the noise. Moreover the strong selective ^{7}Be neutrino suppression, which is the nice feature of the MSW effect, is somehow degraded by the presence of matter noise. Consequently the longstanding conflict between chlorine and Kamiokande data is exacerbated and the data fit gets worse. Indeed, the presence of the matter density noise makes the data fit a little poorer: $\chi^2_{min} = 0.1$ for $\xi = 0$, it becomes $\chi^2_{min} = 0.8$ for $\xi = 4\%$ and even $\chi^2_{min} = 2$ for $\xi = 8\%$.

In conclusion we have shown that the MSW solution exists for any realistic levels of matter density noise ($\xi \leq 4\%$). Moreover the MSW solution is essentially stable in mass ($4 \cdot 10^{-6}$eV$^2 < \delta m^2 < 10^{-5}$eV^2 at 90% CL), whereas the mixing appears more sensitive to the level of fluctuations.

4. Let us also stress the fact that the solar neutrino experiments could be viable tools for providing information on the matter fluctations in the solar center. In particular, the future Borexino experiment [16], aiming to detect the ^{7}Be neutrino flux could be sensitive to the presence of solar matter fluctuations, as the ^{7}Be neutrinos are those mostly affected by the presence of matter noise.

In the relevant MSW paramter region for the noiseless case, the Borexino signal cannot be definitely predicted (see Fig. 2a). Within the present allowed C.L. regions (dotted line) the expected rate, $Z_{Be} = R_{Be}^{pred}/R_{Be}^{SSM}$ (solid lines), is in the range $0.2 \div 0.7$.

On the other hand, when the matter density noise is switched on, e.g. $\xi = 4\%$ (see Fig. 2b), the minimal allowed value for Z_{Be} becomes higher, $Z_{Be} \geq 0.4$. Hence, if the MSW mechanism is responsable for the solar neutrino deficit and Borexino experiment detects a low signal, say $Z_{Be} \lesssim 0.3$ (with good accuracy) this will imply that a 4% level of matter fluctuations in the central region of the sun is unlikely .

Once more, the solar neutrino detection turns

[4] For gallium result we have taken the weighted average of GALLEX datum $R_{Ga}^{exp} = (77 \pm 8 \pm 5)$SNU[13] and SAGE $R_{Ga}^{exp} = (69 \pm 11 \pm 6)$SNU[14].

Fig. 2a

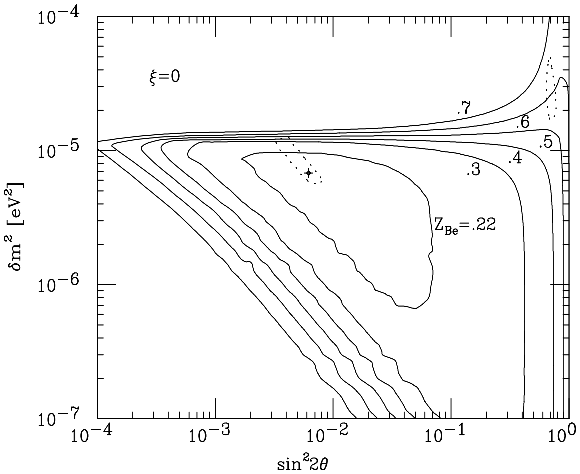
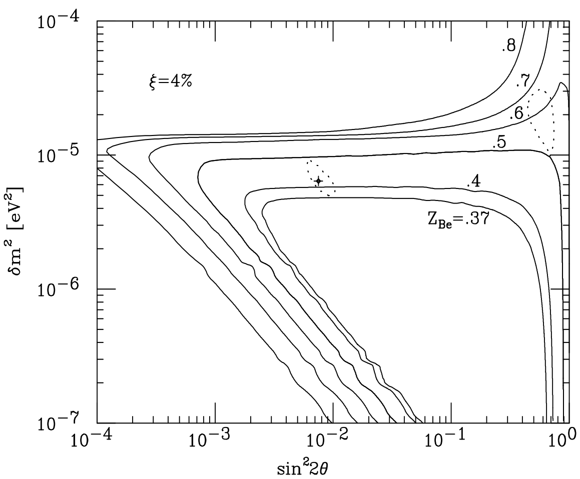


Fig. 2b



out to be an important approach for studying the solar physics.

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